BRL R 1326

BRL

REPORT NO. 1326

DYNAMICS OF LIQUID FILLED SHELL: NON-CYLINDRICAL CAVITY

by

E. H. Wedemeyer

August 1966

[redacted]

U. S. ARMY MATERIEL COMMAND
BALLISTIC RESEARCH LABORATORIES
ABERDEEN PROVING GROUND, MARYLAND

BALLISTIC RESEARCH LABORATORIES

REPORT NO. 1326

AUGUST 1966

[redacted] APPROVED FOR PUBLIC RELEASE [redacted]

DYNAMICS OF LIQUID FILLED SHELL: NON-CYLINDRICAL CAVITY

E. H. Wedemeyer

Exterior Ballistics Laboratory

RDT&E Project No. 1P014501B11A

ABERDEEN PROVING GROUND, MARYLAND

B A L L I S T I C   R E S E A R C H   L A B O R A T O R I E S

REPORT NO. 1326

EHWedemeyer/so
Aberdeen Proving Ground, Md.
August 1966

DYNAMICS OF LIQUID FILLED SHELL: NON-CYLINDRICAL CAVITY

ABSTRACT

A theory is presented for the approximate computation of eigen-frequencies of liquid oscillations in non-cylindrical cavities. The eigen-frequencies are essential for the prediction of stability of liquid-filled shell. The theory reduces the problem of finding the eigen-value to a simple integration which can be performed by hand computation when the shape of the cavity is known. Comparison of theoretical prediction and available experimental data shows very good agreement.

Following page is blank

TABLE OF CONTENTS

Next page is blank

I. INTRODUCTION

The dynamic behavior of liquid-filled, spinning shell can be reliably predicted if the cavity occupied by the liquid is cylindrical or if the cavity is spheroidal and completely filled with liquid. The latter case, a spheroidal cavity completely filled with liquid, is of relatively little practical interest for the shell designer. In fact most cavities are nearly cylindrical and, furthermore, cavities are usually not 100 percent filled. It is, therefore, very fortunate that the case of a partially filled cylindrical cavity is accessible to theoretical analysis.

The theory for this case was published by Stewartson[1*] in 1959 and has since become the most important tool for the design of stable shell. Although most practical cavities are approximately cylindrical, some deviate sufficiently from exact cylindrical shapes that there is some doubt whether treating them as cylinders is still a good approximation. To clarify this point, Karpov[2] has made extensive experimental investigations of non-cylindrical cavities. (All cavities considered are bodies of revolution.) It was found that rounded corners produce very little effect on the range of instability but that considerable changes result from modifications like conical reduction of one or both ends of the cavity. In view of the large variety of cavity shapes, it is not possible to explore the effect of cavity shape solely on an experimental basis.

---

* *Superscript numbers denote references which may be found on page 20.*

On the other hand, an exact theoretical approach to the problem appears to be quite hopeless for the following reason: The equations of the fluid motion are to be solved for boundary conditions that are imposed by the shape of the cavity walls. If the cavity is cylindrical, and only in this case, the solution is separable in radial and axial components. The tremendous simplification gained by the separation of variables is lost if the cavity shape deviates, however slightly, from a perfect cylinder. Solutions could be found, possibly, by numerical methods; however, such an approach is quite impractical considering that the computation cannot be performed prior to the cavity design. The best to be done within the borders of an exact theory is to compute the desired data, e.g., eigen-frequencies - for a comprehensive class of cavity shapes. The shell designer would then approximate the actual cavity by one for which data had been computed and tabulated. However, the advantage of an exact solution is lost when the cavity is only approximate, and one might ask whether an approximate solution for the exact cavity is not preferable.

II. THEORY OF APPROXIMATE EIGEN-FREQUENCIES

Of particular importance for the stability of the shell are the eigen-frequencies or frequencies of free oscillation of the liquid. According to Stewartson [1], instability occurs whenever any of the eigen-frequencies falls within a certain bandwidth about the frequency of nutation of the shell. Details on this stability theory are found in Stewartson's paper. [1] Whether a shell is stable can be predicted when the eigen-frequencies and the residues of the forcing term, which determine the bandwidth, are known. For cylindrical cavities, the eigen-frequencies and residues are computed exactly and given in Stewartson's tables.[1,3] For small deviations from cylindrical shape, small changes of the eigen-frequencies and the residues must be expected. While accurate values for the eigen-frequencies are essential for the prediction of the dynamic behavior, the exact values for the residues are less important, since these determine only the bandwidth and usually the latter is affected by other factors, such as viscosity, more than by deviations from cylindrical shapes. From Karpov's experimental data[2] of un-damping rates, it appears that it is sufficient and safe to assume a value for the residue equal to that of an "equivalent" cylinder, i.e., one with the same volume and height. Thus, we are left with the determination of the eigen-frequencies for non-cylindrical cavities. The following analysis rests on the assumption that the radius of the cavity, a, is a slowly varying function of the distance along the axis, z, i.e.,

$$\left|\frac{da}{dz}\right| \ll 1 \tag{1}$$

From Stewartson's analysis it follows that the oscillatory part of the pressure - which results from the liquid oscillations - is of the form:

$$p = P(r,z)\, e^{i(\omega t-\theta)} \tag{2}$$

where P(r,z) satisfies the differential equation:

$$\frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial P}{\partial r} - \frac{1}{r^2} P = \alpha^2 \frac{\partial^2 P}{\partial z^2} \tag{3}$$

with
$$\alpha^2 = \frac{4}{(1 - \tau)^2} - 1 \tag{4}$$

ω is the eigen-frequency, (r, θ, z) are polar coordinates and τ is the dimensionless eigen-frequency, $\tau = \frac{\omega}{\Omega}$, where Ω is the frequency of spin.

Let us use the notation (u, v, w) for the components of the oscillatory velocity in the polar coordinate system (r,θ,z). Then (u, v, w) can be expressed by certain linear combinations of the pressure and its partial derivatives $\frac{\partial p}{\partial r}$ and $\frac{\partial p}{\partial z}$.

We assume that the cavity shape is given by:

$$a = a(z) , \qquad 0 \le z \le 2c \tag{5}$$

and that Equation (1) holds for $0 \le z < 2c$. If the cavity is partially filled with liquid it is assumed, as in Stewartson's case, that the undisturbed free surface is a cylinder of radius b. For the following analysis we must assume that

$$b < a(z) \quad \text{for} \quad 0 \le z \le 2c \tag{6}$$

Equation (3) must be solved in connection with certain boundary conditions, e.g.,

$$u = \frac{da}{dz} w \quad \text{at} \quad r = a(z) \tag{7}$$

$$w = 0 \quad \text{at} \quad z = 0,\ 2c \tag{8}$$

and one boundary condition at $r = b$, which requires that the pressure be zero on the free surface when the cavity is partially filled. For a better understanding, a brief review of the eigen-value problem for Stewartson's case is given in the following: For the cylindrical cavity, i.e., if $\frac{da}{dz} = 0$, the solution of (3) with appropriate boundary conditions can be found by separation of variables:

$$P = C(\alpha kr)\ .\quad \cos kz \tag{9}$$

where $C = A \cdot J_1(\alpha kr) + B \cdot Y_1(\alpha kr)$ and J_1 , Y_1 are Bessel functions of the first and second kind with the argument αkr . The solution (9) must satisfy certain boundary conditions at $r = b$ and $r = a$. Without giving a detailed derivation (which can be found in Reference 1) it suffices here to state the following results: For given b^2/a^2 and τ the boundary conditions at $r = b$ and $r = a$ lead to a transcendental equation for ka with discrete roots:

$$ka = \eta_n\ (b^2/a^2\ ,\ \tau)\ ,\quad n = 1,\ 2,\ 3,\ \ldots. \tag{10}$$

where n is related to the number of radial waves of the solution (9).

The boundary conditions at $z = 0$, $2c$ lead to a similar equation for kc with roots:

$$kc = \frac{\pi}{2}\ [2j+1]\ j = 0,\ 1,\ 2\ \ldots \tag{11}$$

Since w is proportional to $\frac{\partial p}{\partial z}$, Equation (8) can be written:

$$\frac{\partial p}{\partial z} = 0 \quad \text{at} \quad z = 0,\ 2c \quad . \tag{12}$$

Equations (12) and (9) yield:

$$\sin k\, 2c = 0\ , \qquad kc = \frac{\pi}{2} \cdot m \tag{13}$$

Because of certain symmetry requirements, the number of half-waves in the z-direction must be odd, i.e., $m = (2j+1)$ and Equation (11) follows from (13). Comparison of (11) and (10) gives a condition for the fineness ratio:

$$\frac{c}{a[2j+1]} = \frac{\pi/2}{\eta_n(b^2/a^2, \tau)} \tag{14}$$

Equation (14) expresses that only certain discrete fineness ratios $\frac{c}{a}$ exist for given b^2/a^2 and τ or, for given b^2/a^2 and c/a there exists a set of discrete eigen-functions τ_{jn} according to the choice of j and n in Equation (14). In Stewartson's tables[1] the various $\frac{c}{a(2j+1)}$ are tabulated as functions of b^2/a^2 and τ. Let us call these functions $S_n(b^2/a^2, \tau)$, i.e.,

$$\frac{c}{a(2j+1)} = S_n(b^2/a^2, \tau) = \frac{\pi/2}{\eta_n(b^2/a^2, \tau)} \tag{15}$$

Returning to the eigen-value problem for the non-cylindrical cavity, we notice that the only difference is in the boundary condition (7). This condition states that the velocity component normal to the wall $r = a$ is zero. If now the inclination of the wall toward the axis is small, i.e., $\left|\frac{da}{dz}\right| \ll 1$, an obvious approximation of Equation (7) is:

$$u = 0 \quad \text{at} \quad r = a(z) \tag{16}$$

The form of the approximate boundary condition (16) suggests searching for approximate solutions of Equation (3) which are of the same general form as Equation (9) except that the radial wave number, k, should be a slowly varying function of z according to the fact that the radius, a, changes slowly with z. Thus, we try as an approximate solution of (3):

$$P = C(\alpha kr) \; . \; \cos \Phi(z) \tag{17}$$

where C is a cylinder function of the argument αkr and k depends weakly on z . The phase $\Phi(z)$ can no longer be assumed to be equal to kz but rather $\Phi = \int_0^z k d\zeta$.

It can be verified easily that (17) approximately satisfies the differential Equation (3) provided that:

$$k(z) = \frac{d\Phi}{dz} \tag{18}$$

and

$$\left|\frac{dk}{dz}\right| \ll k^2 \tag{19}$$

Since the boundary condition at $r = b$ and the approximate boundary condition at $r = a$ (Equation (16)) are - locally - the same as for the cylindrical cavity, we arrive naturally at the same condition for ka, viz., according to Equation (10),

$$k = \frac{1}{a} \eta_n \ (b^2/a^2, \tau) \tag{20}$$

Equation (20) gives the z- dependent wave number $k(z)$ and, at the same time, shows that the condition (19) is a consequence of $\left|\frac{da}{dz}\right| \ll 1$ (since η_n and its derivative with respect to b^2/a^2 are of order unity).

Thus, we conclude that (17) with (18) approximately satisfies the differential Equation (3) <u>and</u> the boundary conditions at $r = b$ and $r = a$.

Finally, the boundary condition (12): $\frac{\partial p}{\partial z} = 0$ at $z = 0, 2c$ requires that $\sin \Phi = 0$ at $z = 0, 2c$ or:

$$\Phi(0) = 0 \tag{21}$$

$$\Phi(2c) = \int_0^{2c} k(z)\, dz = \pi[2j+1] \tag{22}$$

Equation (22) can be considered as a generalization of Equation (11). Substituting (20) into (22) gives:

$$\pi[2j+1] = \int_0^{2c} \frac{1}{a} \eta_n (b^2/a^2, \tau)\, dz \tag{23}$$

Equation (23) connects b and τ, i.e., if the radius of the cylindrical void, b, is given, (23) determines τ and vice versa.

According to Equation (15) the function η_n is related to S_n, a function which is tabulated in Stewartson's tables. Thus, it is convenient to rewrite (23) by substituting η_n according to Equation (15) into (23) and, after dividing by $\pi(2j+1)$, one obtains:

$$1 = \frac{1}{2c} \int_0^{2c} \frac{c/a(2j+1)}{S_n\,(b^2/a^2\,,\ \tau)}\, dz \tag{24}$$

Equation (24) is easy to memorize: $S_n\,(b^2/a^2, \tau)$ is just the $c/a(2j+1)$-value of a cylinder with eigen-frequency τ and fill-ratio b^2/a^2. Thus, the integrand of (24) is the ratio of the local $c/a(2j+1)$ - value and the $c/a(2j+1)$ - value which would correspond to τ and the local b^2/a^2 - value. Equation (24) then states that the mean value of this ratio - averaged over the length of the cavity - should be equal to one.

Usually, the higher radial modes are unimportant, so that (24) must be solved only for $n = 1$. We will therefore write just S instead of S_n, having in mind that (24) is valid for any of the S_n values.

The evaluation of Equation (24) is difficult when S is given numerically (as in Stewartson's tables), since one has to assume both, b and τ, in order to perform the integration numerically and eventually repeat the procedure with changed values of τ (or b) until the correct value of τ (or b) can be observed by interpolation. For a 100 percent filled cavity, i.e., $b = 0$, the integration simplifies considerably, since S becomes independent of z and can be taken out of the integral. One obtains from (24):

$$S(0, \tau) = \frac{1}{2c} \int_0^{2c} \frac{c}{a(2j+1)}\, dz \tag{25}$$

Since S is the $\frac{c}{a(2j+1)}$ - value of a cylinder with eigen-frequency τ, Equation (25) can be interpreted in the following way: A completely filled non-cylindrical cavity has the same eigen-frequencies τ_{nj} as an "equivalent cylindrical cavity", which is defined by having a fineness ratio, c/a equal to the averaged c/a of the non-cylindrical cavity. For a 100 percent filled cavity, therefore, only one integration must be performed to determine the average c/a.

One way of solving the eigen-value problem given in Equation (24) is to approximate $1/S$ by a power series in b^2/a^2. If τ is given and b is to be determined one could plot $1/S$, for the particular τ given, versus b^2/a^2 and approximate the obtained curve by a polynomial in b^2/a^2. If neither τ nor b^2/a^2 are too large, the following formula is convenient and quite accurate:

$$\frac{1}{S} = \frac{1}{S_0} \left[1 + 1.26 \left(\frac{b^2}{a^2}\right)^2\right] \tag{26}$$

S_0 is the value $S(0, \tau)$ as obtained from Stewartson's tables for $b^2/a^2 = 0$. The approximation (26) is valid within the following limits:

$$\begin{gathered} 0 \leq \tau \leq 0.12 \\ 0 \leq b^3/a^2 \leq 0.15 \end{gathered} \tag{27}$$

with (26) substituted into (24) one obtains:

$$S_o(\tau) = \frac{1}{2c} \int_0^{2c} \frac{c}{a(2j+1)} \left[1 + 1.26\left(\frac{b^2}{a^2}\right)^2\right] dz \tag{28}$$

Equation (28) has the advantage that the right side is independent of τ and the left side independent of b, i.e., by integration of (28) one finds a relation of the form:

$$S_o(\tau) = C_1 + C_2 \, b^4 \tag{29}$$

A better approximation than (28) may be obtained by choosing a coefficient different from 1.26 in Equation (26) adapted to the particular τ, if τ is known. One could also include higher powers of b^2/a^2.

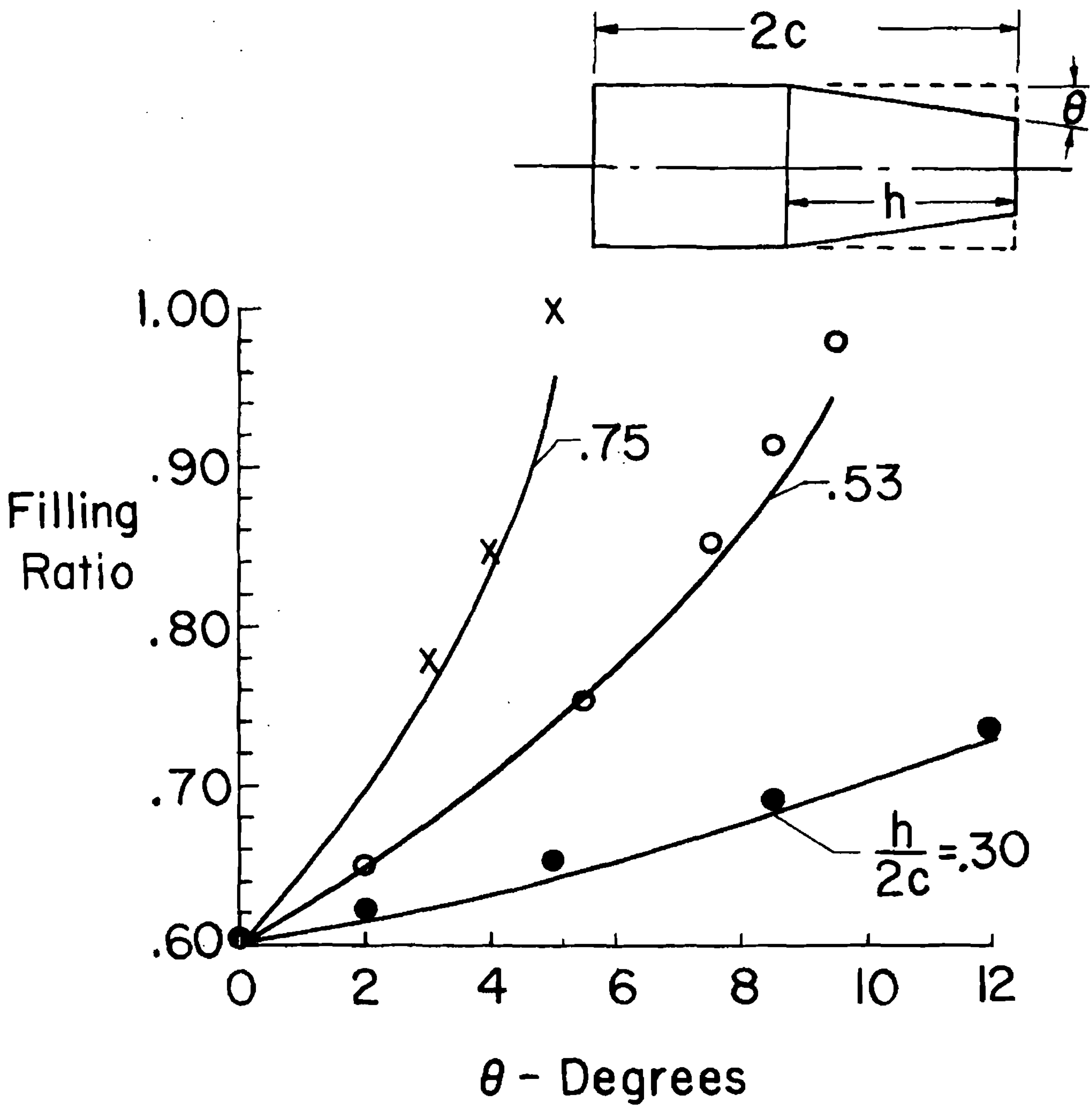
III. COMPARISON WITH EXPERIMENTS

In order to test the theory given in the preceeding section some of the experimental findings of Karpov[2] have been compared with theoretical predictions. The cavities investigated by Karpov were composed of a cylindrical and a conical section. A schematic of a cavity shape is shown in the following figure. In the diagram, the filling ratio which corresponds to a <u>fixed</u> eigen-frequency τ_0 is plotted versus the variable cone-angle θ. Three curves are shown that correspond to three different ratios of conical section h to overall length $2c$. For $\theta = 0$ all cavities degenerate into a cylinder of fineness ratio $\frac{c}{a} = 2.687$ and the filling ratio $(1 - b^2/a^2)$ attains Stewartson's value. (It is $j = 1$, $n = 1$). The solid curves in the figure show the theoretically predicted values, the symbols the experimental data. The theory obviously gives good predictions even for θ-values for which the supposition on which the theory rests, viz. $\left|\frac{da}{dz}\right| = \ll 1$, is hardly fulfilled. The reason for this is, roughly, that the neglected terms in the boundary conditions and in the differential equation are orthogonal, or nearly so, to the other terms. Therefore, although the "local error" is of order $\left|\frac{da}{dz}\right|$, the "averaged error" over the length of the cavity can be very small. The theoretical curves in the figure were obtained by solving Equation (25) for fixed $\tau = 0.055$. To perform the integration, the following approximation was used:

$$\frac{1}{s} = 0.944 + 1.074 \left(\frac{b^2}{a^2}\right)^2$$

The integration could then be performed analytically and b^2/a^2 is found as a function of θ and $\frac{h}{2c}$.

Filling Ratio At Fixed Resonance Frequency $\tau_0 = \tau_n = 0.055$



REFERENCES

1. Stewartson, K. On the Stability of a Spinning Top Containing Liquid. J. Fluid Mech., 5, Part 4, 1959.

2. Karpov, B. G. Dynamics of Liquid Filled Shell: Resonance in Modified Cylindrical Cavities. Ballistic Research Laboratories Report No. , 1966.

3. Karpov, B. G. Dynamics of Liquid Filled Shell: Aids for Designers. Ballistic Research Laboratories Memorandum Report No. 1477, May 1963.

DISTRIBUTION LIST

No. of Copies	Organization
20	Commander Defense Documentation Center ATTN: TIPCR Cameron Station Alexandria, Virginia 22314
1	Director of Defense Research and Engineering (OSD) ATTN: Asst Dir/Tac Msl & Ord Washington, D.C. 20301
1	Director Defense Atomic Support Agency Washington, D.C. 20301
1	Commanding General U.S. Army Materiel Command ATTN: AMCRD-RP-B Washington, D.C. 20315
2	Commanding General U.S. Army Missile Command Redstone Arsenal, Alabama 35809
1	Commanding Officer U.S. Army Engineer Research & Development Laboratories ATTN: STINFO Div Fort Belvoir, Virginia 22060
1	Commanding Officer U.S. Army Edgewood Arsenal ATTN: SMUEA-R Edgewood Arsenal, Maryland 21010
1	Commanding Officer Fort Detrick Frederick, Maryland 21701
2	Commanding Officer U.S. Army Frankford Arsenal ATTN: Lib Br., 0270, Bldg 40 Philadelphia, Pennsylvania 19137

No. of Copies	Organization
3	Commanding Officer U.S. Army Picatinny Arsenal Dover, New Jersey 07801
1	Commanding Officer U.S. Army Materials Research Agency ATTN: Mr. J. Bluhm Watertown, Massachusetts 02172
1	Commanding Officer U.S. Army Watertown Arsenal Watertown, Massachusetts 02172
1	Commanding General U.S. Army Combat Developments Command Fort Belvoir, Virginia 22060
1	Commanding General U.S. Army Combat Developments Command Combat Arms Group Fort Leavenworth, Kansas 66027
1	Director U.S. Army Research Office 3045 Columbia Pike Arlington, Virginia 22204
1	Superintendent U.S. Military Academy ATTN: Prof of Ord West Point, New York 10996
3	Commander U.S. Naval Air Systems Command Headquarters ATTN: AIR-604 Department of the Navy Washington, D.C. 20360
1	Commanding Officer U.S. Naval Air Development Center, Johnsville Warminster, Pennsylvania 18974

DISTRIBUTION LIST

No. of Copies	Organization
2	Commander U.S. Naval Missile Center Point Mugu, California 93041
2	Commander U.S. Naval Ordnance Laboratory Silver Spring, Maryland 20910
1	Commander U.S. Naval Ordnance Test Station ATTN: Code 753 China Lake, California 93557
1	Commander U.S. Naval Weapons Laboratory Dahlgren, Virginia 22448
1	Commanding Officer U.S. Naval Weapons Station ATTN: Tech Lib Seal Beach, California 90740
1	Commanding Officer & Director David W. Taylor Model Basin ATTN: Aerodynam Lab Washington, D.C. 20007
1	Superintendent U.S. Naval Postgraduate School ATTN: Tech Rpt Sec Monterey, California 93940
1	Chief of Naval Research Department of the Navy Washington, D.C. 20360
1	HQ, USAF (AFRDC) Washington, D.C. 20330
1	AEDC (AER) Arnold AFS Tennessee 37389
1	RTD (RTTM) Bolling AFB, D.C. 20332
1	AFFDL (FDM) Wright-Patterson AFB Ohio 45433
1	U.S. Atomic Energy Commission Gmelin Institute ATTN: Depository Library 7 Woodland Avenue Larchmont, New York 10538
1	Headquarters National Aeronautics and Space Administration Washington, D.C. 20546
1	Director NASA Scientific and Technical Information Facility ATTN: SAK/DL P.O. Box 33 College Park, Maryland 20740
1	Director National Aeronautics and Space Administration Ames Research Center ATTN: Mr. H. J. Allen Moffett Field, California 94035
3	Director Jet Propulsion Laboratory ATTN: Rpt Gp (2 cys) Mr. Jack Lowell 4800 Oak Grove Drive Pasadena, California 91103

DISTRIBUTION LIST

No. of Copies	Organization
1	Director National Aeronautics and Space Administration George C. Marshall Flight Center ATTN: R-AERO-DAM, Miss Bauer Huntsville, Alabama 35812
2	Director Applied Physics Laboratory The Johns Hopkins University 8621 George Avenue Silver Spring, Maryland 20910
1	Aerojet-General Corporation 6352 North Irwindale Road Azusa, California 91703
1	Bendix Aviation Corporation Bendix Mishawaka Division 400 South Beiger Street Mishawaka, Indiana 46544
1	Boeing Scientific Research Laboratory ATTN: Dr. Y. H. Pao P. O. Box 3981 Seattle, Washington 98124
1	Curtiss-Wright Corporation Wright Aeronautical Division ATTN: Sales Dept (Govt) Wood-Ridge, New Jersey 07070
1	Douglas Aircraft Company, Inc. 3000 Ocean Park Boulevard Santa Monica, California 90405
1	Eastman Kodak Company Rochester, New York 14604

No. of Copies	Organization
1	Fairchild Hiller Republic Aviation Division ATTN: Mil Contr Dept Farmingdale, New York 11735
1	Fairchild Stratos Corporation ATTN: Lib Wyandanch, New York 11798
1	General Dynamics/Pomona ATTN: Guided Msl Div 1675 West Fifth Avenue P. O. Box 2507 Pomona, California 91766
1	Goodyear Aerospace Corporation 1210 Massilon Road Akron, Ohio 44315
1	Grumman Aircraft Corporation Bethpage, New York 11714
1	Hughes Aircraft Company Florence Avenue and Teal Street Culver City, California 90232
1	Ling-Temco-Vought, Inc. P. O. Box 5907 Dallas, Texas 75222
1	Arthur D. Little, Inc. 15 Acorn Park Cambridge, Massachusetts 02140
1	Marquardt Aircraft Company ATTN: Mr. R. Marquardt 16555 Saticoy Street Van Nuys, California 91404
1	The Martin Company Baltimore, Maryland 21203

DISTRIBUTION LIST

No. of Copies	Organization
1	McDonnell Aircraft Corporation P. O. Box 516 St. Louis, Missouri 63103
3	North American Aviation, Inc. ATTN: Aerophys Lib 12214 Lakewood Boulevard Downey, California 90241
1	Northrop Corporation NORAIR Division 1001 East Broadway Hawthorne, California 90250
1	Raytheon Company ATTN: Guided Msl & Rdr Div Lexington, Massachusetts 02173
2	Sandia Corporation ATTN: Info Distr Div P.O. Box 5800 Albuquerque, New Mexico 87115
1	Sperry Gyroscope Company Division of the Sperry Rand Corporation ATTN: Librarian Great Neck, New York 11020
1	TRW Systems Group ATTN: Tech Lib One Space Park Redondo Beach, California 90278
1	United Aircraft Corporation Missile and Space Division East Hartford, Connecticut 06118
1	Cornell Aeronautical Laboratory, Inc. ATTN: Lib P.O. Box 235 Buffalo, New York 14221
3	IIT Research Institute ATTN: Doc Lib Chicago, Illinois 60616
1	The Johns Hopkins University Project THOR 3506 Greenway Baltimore, Maryland 21218
2	Massachusetts Institute of Technology ATTN: Guided Msl Lib 77 Massachusetts Avenue Cambridge, Massachusetts 02139
1	University of Michigan Willow Run Laboratories P.O. Box 2008 Ann Arbor, Michigan 48104
1	Professor G. F. Carrier Division of Engineering & Applied Physics Harvard University Cambridge, Massachusetts 02138
1	Professor E. J. McShane Department of Mathematics University of Virginia Charlottesville, Virginia 22901
1	Dr. Norman Abramson Southwest Research Institute 8500 Culebra Road San Antonio, Texas 78206
1	Dr. C. P. Boner Defense Research Laboratory University of Texas P.O. Box 8029 500 East 24th Street Austin, Texas 78712

DISTRIBUTION LIST

No. of Copies	Organization
1	Dr. B. G. Karpon P.O. Box 306 Tryon, North Carolina 28782
1	Dr. J. Siekmann College of Engineering University of Florida Gainesville, Florida 32603
1	Dr. M. K. Zucrow Purdue University Lafayette, Indiana 47907

Aberdeen Proving Ground

Ch, Tech Lib

Air Force Ln Ofc
Marine Corps Ln Ofc
Navy Ln Ofc
CDC Ln Ofc

Unclassified

Security Classification

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY *(Corporate author)* U.S. Army Ballistic Research Laboratories Aberdeen Proving Ground, Maryland	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

3. REPORT TITLE

DYNAMICS OF LIQUID FILLED SHELL: NON-CYLINDRICAL CAVITY

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

5. AUTHOR(S) *(Last name, first name, initial)*

Wedemeyer, Erich H.

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
August 1966	25	3

8a. CONTRACT OR GRANT NO. b. PROJECT NO. 1P014501B11A c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) Report No. 1326 9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*

10. AVAILABILITY/LIMITATION NOTICES This document is subject to special export controls and each transmittal to foreign governments or foreign nationals may be made only with prior approval of Commanding Officer, U.S. Army Ballistics Research Laboratories, Aberdeen Proving Ground, Maryland.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	U.S. Army Materiel Command Washington, D. C.

13. ABSTRACT

A theory is presented for the approximate computation of eigen-frequencies of liquid oscillations in non-cylindrical cavities. The eigen-frequencies are essential for the prediction of stability of liquid-filled shell. The theory reduces the problem of finding the eigen value to a simple integration which can be performed by hand computation when the shape of the cavity is known. Comparison of theoretical prediction and available experimental data shows very good agreement.

DD FORM 1 JAN 64 1473

Unclassified

Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Liquid Filled Shell Stability Non-Cylindrical Cavity						

INSTRUCTIONS

1. ORIGINATING ACTIVITY: Enter the name and address of the contractor, subcontractor, grantee, Department of Defense activity or other organization *(corporate author)* issuing the report.

2*a*. REPORT SECURITY CLASSIFICATION: Enter the overall security classification of the report. Indicate whether "Restricted Data" is included. Marking is to be in accordance with appropriate security regulations.

2*b*. GROUP: Automatic downgrading is specified in DoD Directive 5200.10 and Armed Forces Industrial Manual. Enter the group number. Also, when applicable, show that optional markings have been used for Group 3 and Group 4 as authorized.

3. REPORT TITLE: Enter the complete report title in all capital letters. Titles in all cases should be unclassified. If a meaningful title cannot be selected without classification, show title classification in all capitals in parenthesis immediately following the title.

4. DESCRIPTIVE NOTES: If appropriate, enter the type of report, e.g., interim, progress, summary, annual, or final. Give the inclusive dates when a specific reporting period is covered.

5. AUTHOR(S): Enter the name(s) of author(s) as shown on or in the report. Enter last name, first name, middle initial. If military, show rank and branch of service. The name of the principal author is an absolute minimum requirement.

6. REPORT DATE: Enter the date of the report as day, month, year; or month, year. If more than one date appears on the report, use date of publication.

7*a*. TOTAL NUMBER OF PAGES: The total page count should follow normal pagination procedures, i.e., enter the number of pages containing information.

7*b*. NUMBER OF REFERENCES: Enter the total number of references cited in the report.

8*a*. CONTRACT OR GRANT NUMBER: If appropriate, enter the applicable number of the contract or grant under which the report was written.

8*b*, 8*c*, & 8*d*. PROJECT NUMBER: Enter the appropriate military department identification, such as project number, subproject number, system numbers, task number, etc.

9*a*. ORIGINATOR'S REPORT NUMBER(S): Enter the official report number by which the document will be identified and controlled by the originating activity. This number must be unique to this report.

9*b*. OTHER REPORT NUMBER(S): If the report has been assigned any other report numbers *(either by the originator or by the sponsor)*, also enter this number(s).

10. AVAILABILITY/LIMITATION NOTICES: Enter any limitations on further dissemination of the report, other than those imposed by security classification, using standard statements such as:

(1) "Qualified requesters may obtain copies of this report from DDC."

(2) "Foreign announcement and dissemination of this report by DDC is not authorized."

(3) "U. S. Government agencies may obtain copies of this report directly from DDC. Other qualified DDC users shall request through ________________."

(4) "U. S. military agencies may obtain copies of this report directly from DDC. Other qualified users shall request through ________________."

(5) "All distribution of this report is controlled. Qualified DDC users shall request through ________________."

If the report has been furnished to the Office of Technical Services, Department of Commerce, for sale to the public, indicate this fact and enter the price, if known.

11. SUPPLEMENTARY NOTES: Use for additional explanatory notes.

12. SPONSORING MILITARY ACTIVITY: Enter the name of the departmental project office or laboratory sponsoring *(paying for)* the research and development. Include address.

13. ABSTRACT: Enter an abstract giving a brief and factual summary of the document indicative of the report, even though it may also appear elsewhere in the body of the technical report. If additional space is required, a continuation sheet shall be attached.

It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as *(TS)*, *(S)*, *(C)*, or *(U)*.

There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

14. KEY WORDS: Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of links, rules, and weights is optional.